HIFAN 1749

A PULSED ELECTRIC LENS FOR NDCX

by
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①

A pulsed electric lens for NDCX

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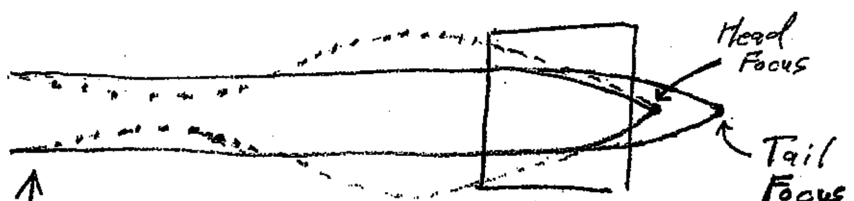
- To compress pulse, $v_{Tail} > v_{Head}$
- This causes a chromatic aberration:



- Place Fast Pulsed Lens here to compensate velocity "Tilt"

②

Considerations

Time scale is ~ pulse length ~ 1.0 μs

Lens works only in vacuum

Lens must be compact ($\lesssim$ 30 cm)

Voltages $\lesssim$ 100 kV

Programable waveform

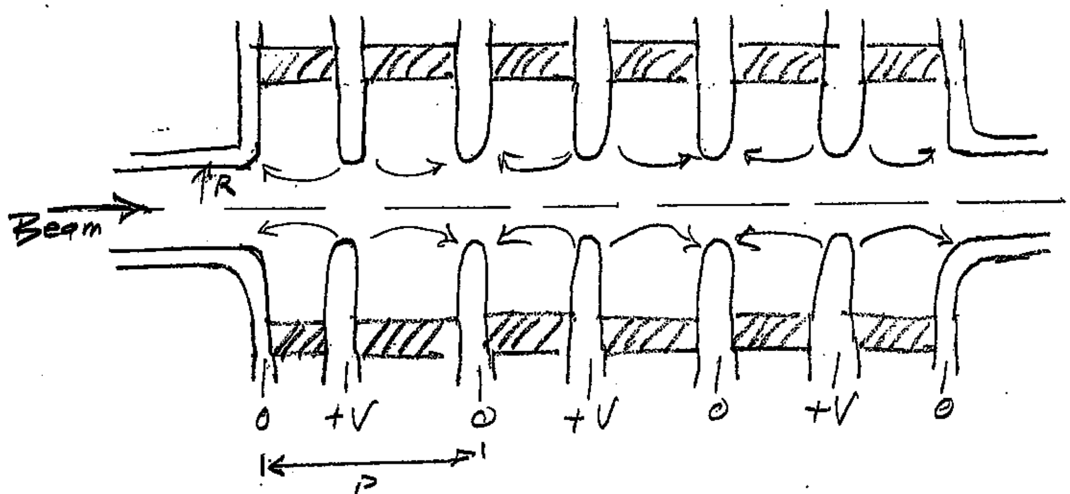
Reasonable cost – look at energy/power

Solenoid $\gtrsim$ 1.0 kJ, 10^9 watts

Electric lens $\gtrsim$ 30 mJ, 30 kW

(3)

Axisymmetric multigap lens system



$$\phi(R,z,t) = \begin{pmatrix}\text{Potential at} \\ r = R\end{pmatrix} \simeq \frac{V(t)}{2}\left(1 - \cos\left(\frac{2\pi z}{P}\right)\right) \quad \text{inside}$$

$$= 0 \quad \text{outside}$$

Length $= nP$

$n = \#$ of periods (= 3 in drawing)

(4)

Potential inside the bore ($r < R$)

$$\nabla^2 \phi = 0 \qquad (\vec{E} = -\vec{\nabla}\phi)$$

$$\rightarrow \quad \phi(r,z,t) \simeq \phi_0(z,t) - \frac{\partial^2 \phi_0}{\partial z^2}\frac{r^2}{4}$$

$\phi_0(z,t)$ = on-axis potential

$\therefore$ Solve $\displaystyle \frac{\partial^2 \phi_0}{\partial z^2} - \frac{4}{R^2}\phi_0 = -\frac{4}{R^2}\phi(R,z,t)$

$$= -\frac{4}{R^2}\frac{V(t)}{2}\left(1 - \cos\left(\frac{2\pi z}{P}\right)\right) \quad \text{(inside)}$$

$$\left.\begin{aligned} E_z &\cong -\frac{\partial \phi_0}{\partial z} \\ E_r &\cong \frac{\partial^2 \phi_0}{\partial z^2}\frac{r}{2} \end{aligned}\right\} \text{near axis values}$$

— This can be done analytically —

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Solve for ion orbits

$$\frac{dz(t)}{dt} = v(t)$$

$$\frac{dv(t)}{dt} = -\frac{qe}{m}\frac{\partial \phi_0}{\partial z}(z(t),t)$$

$$\frac{dx(t)}{dt} = \dot{x}(t)$$

$$\frac{d\dot{x}(t)}{dt} = \frac{qe}{m}\frac{1}{2}\frac{\partial^2 \phi_0}{\partial z^2}(z(t),t)\, x(t)$$

$\left\{ \begin{array}{l} q = \text{charge state} \\ m = \text{mass} \\ e = |\text{electron charge}| \end{array} \right.$

Worked case: $V(t) = (100\,kV)\sqrt{\dfrac{t}{1.0\,\mu s}}$

$n = 4$ periods

$R = 2$ cm

$P = 6$ cm

300 keV K^+ ($m = 38.96$ amu)

(6)


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In[22]:= (*pulsed ES lens 7/15/07*)

         (*input period#, Period length, bore radius*)
         n = 4;
         P = .06;
         R = .02;

         (*input electrode potential vs t*)
         V[t_] = 1.*(10^5)*(t/(10^-6))^.5;

         (*define electrode functions*)
         EE[t_] = V[t]/2/(1+(P/Pi/R)^2);
         DD[t_] = -EE[t]*Exp[-n*P/R];
         AA[t_] = -DD[t]*Sinh[n*P/R];
         CC[t_] = -(P/Pi/R)^2*EE[t];

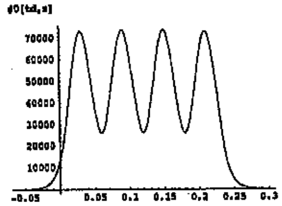
         (*compute on-axis potential and its z-derivatives*)
         φ0[t_, z_] = AA[t]*Exp[2*z/R]*If[z ≤ 0, 1, 0] +
            (V[t]/2 + CC[t]*Cos[2*Pi*z/P] + DD[t]*Cosh[2/R*(z - n*P/2)])*
             If[0 ≤ z ≤ n*P, 1, 0] + AA[t]*Exp[-2/R*(z - n*P)]*If[z ≥ n*P, 1, 0];

         φ0p[t_, z_] = 2/R*AA[t]*Exp[2/R*z]*If[z ≤ 0, 1, 0] +
            (-2*Pi/P*CC[t]*Sin[2*Pi*z/P] + 2/R*DD[t]*Sinh[2/R*(z - n*P/2)])*
             If[0 ≤ z ≤ n*P, 1, 0] - 2/R*AA[t]*Exp[-2/R*(z - n*P)]*If[z ≥ n*P, 1, 0];

         φ0pp[t_, z_] = 4/R^2*AA[t]*Exp[2/R*z]*If[z ≤ 0, 1, 0] +
            4/R^2*(EE[t]*Cos[2*Pi*z/P] + DD[t]*Cosh[2/R*(z - n*P/2)])*
             If[0 ≤ z ≤ n*P, 1, 0] + 4/R^2*AA[t]*Exp[-2/R*(z - n*P)]*If[z ≥ n*P, 1, 0];

         (*plot potentials and z-derivitives at display time td*)
         td = 10^-6;
         Print["φ0[td,z]"]
         Plot[φ0[td, z], {z, -P, (n+1)*P}];
         Print["φ0p[td,z]"]
         Plot[φ0p[td, z], {z, -P, (n+1)*P}];
         Print["φ0pp[td,z]"]
         Plot[φ0pp[td, z], {z, -P, (n+1)*P}];
         Print["φ[td,R,z]"]
         Plot[φ0[td, z] - R^2/4*φ0pp[td, z], {z, -P, (n+1)*P}];
```

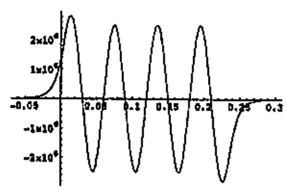
φ0[td,z]



$z \rightarrow$

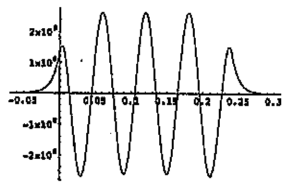
On-Axis Potential at 1.0 μs

ϕ0p[td,z]



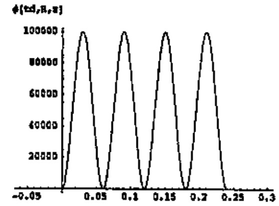
On-axis
$(-E_z)$

ϕ0pp[td,z]



$\sim E_r$

ϕ[td,R,z]



ϕ at aperture

$t = 1.0\,\mu s$ for all

In[361]:=

```
(*input electronic charge and ion mass*)
e = 1.602*(10^-19);
M = 38.96 / (6.022*10^26);

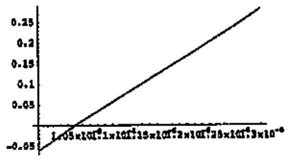
(*input ion values: charge state (q), initial KE in eV (w0),
 initial position (x0,z0),initial angle (xp0),initial and final times (t0,tm)*)
q = 1;
w0 = 3*(10^3);
z0 = -7;
x0 = .01;
xp0 = 0.0;
t0 = 10*10^-7;
tm = 13.0*10^-7;

(*compute initial velocities (v0 and xdot0)*)
v0 = (2*e*w0/M)^.5;
xdot0 = xp0*v0;

(*compute ion trajectory*)
sol = NDSolve[{z'[t] == v[t], v'[t] == -q*e/M*ϕ0p[t, z[t]],
    x'[t] == xdot[t], xdot'[t] == q*e/2/M*ϕ0pp[t, z[t]]*x[t], z[t0] == z0,
    v[t0] == v0, x[t0] == x0, xdot[t0] == xdot0}, {z, v, x, xdot}, {t, t0, tm}];
zz[t_] = z[t] /. sol;
vv[t_] = v[t] /. sol;
xx[t_] = x[t] /. sol;
xxdot[t_] = xdot[t] /. sol;

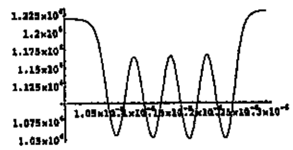
(*plot trajectory functions*)
Print["z[t]"]
Plot[zz[t], {t, t0, tm}];
Print["v[t]"]
Plot[vv[t], {t, t0, tm}];
Print["x[t]"]
Plot[xx[t], {t, t0, tm}];
Print["dx/dz[t]"]
Plot[xxdot[t]/vv[t], {t, t0, tm}];
Print["final dx/dz and focal length (=-x0/(dx/dz)"]
xxdot[tm]/vv[tm]
(-x0)/%
```

z[t]



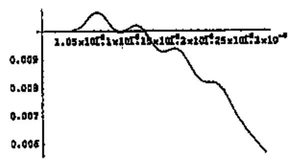
z(t)

v[t]



v(t)

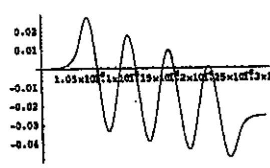
x[t]



x(t)

dx/dz[t]

ion starts at 1.0 μs



x'(t)

final dx/dz and focal length (=-x0/(dx/dz))

Out[386]= {-0.0257209}

Out[387]= {0.388788}

(11)

To find a focal length start ions at various times with $x = 1.0$ cm, $\frac{dx}{dz} = 0$.

$$f = \frac{x_{initial}}{\left(-\frac{dx}{dz}\right)_{final}} = \frac{x_{initial}}{\left(-\dot{x}/v\right)_{final}}$$

initial time	final dx/dz	focal length
0	-.00321	3.12 m
.2 μs	-.00716	1.40
.4	-.01162	.860
.6	-.0163	.615
.8	-.0210	.477
1.0	-.0257	.389